



Maths @ KVS



Maths Policy



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Addition



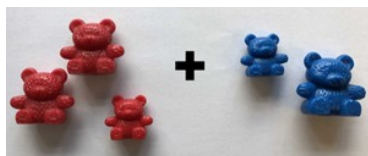
EYFS

Concrete

Counting objects from a group.



Sorting objects into two groups.



Counting two groups of objects.



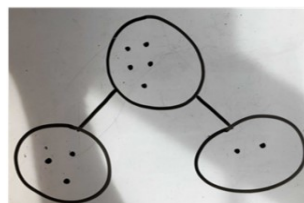
First count the red bears, 1,2,3 then the blue bears 4,5..

Pictorial

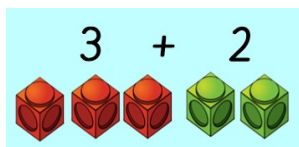
Counting on from the larger number using a number line.



Showing a simple addition using a part, part, whole model. The whole being the total of the two parts.

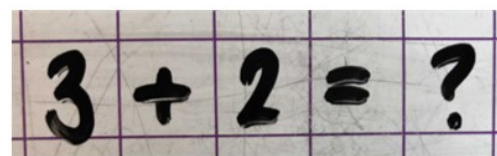


Beginning to make connections between numbers and amounts of objects or images.

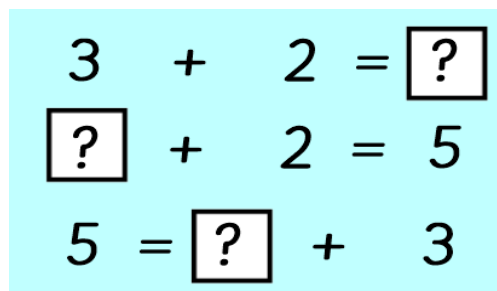


Abstract

Simple number sentences involving two 1 digit numbers.



Variation ideas using two 1 digit numbers.



Teaching notes

Counting

Physically move objects while counting 1 number at a time to ensure accuracy.

Using fingers to count numbers or objects to 10. Use feet (as 10) and fingers to count numbers between 11 and 20.

Using a number line to assist counting on.

Adding

Count the larger group first. Then count on from the first amount to find the total.

When adding two numbers 'lock' the larger number in your head then count on the same number of times as the smaller number.

E.g. 3 in your head, count on two times, 4...5... total is 5.

Number facts

Children begin to learn number bonds to 10. Near number facts one more and one less as well as doubles.

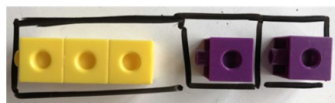
Addition



Year 1

Concrete

Children encouraged not to re-count the larger group.

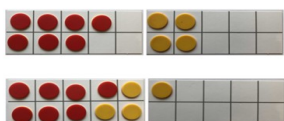


Counting in ones from the larger group or number..

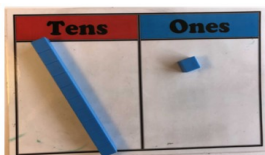


Making each number on a ten frame. Fill the first frame to make 10 and 'some more'.

$$7 + 4 = ?$$

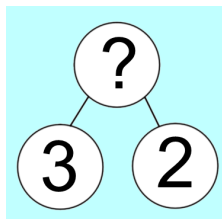


Make numbers using base 10 and place value charts. Showing number of tens and number of ones..

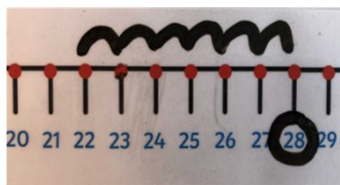


Pictorial

Use part-whole model to represent a number sentence visually to help show the unknown part or whole.



'Lock' the larger number in your head and count on in ones. Use a number line or 100 square to support the counting on.

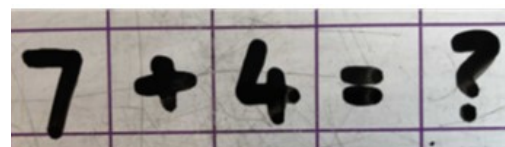


$$22 + 6 = ?$$

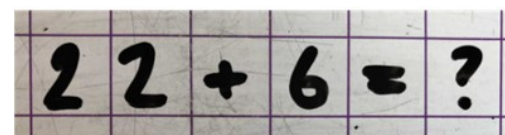


Abstract

Simple number sentences involving two 1 digit numbers that now include crossing over 10.



Number sentences involving a 2 digit number and 1 digit number.



Variation ideas to represent bonds to 10..

5	+		=	10
	+	4	=	10
7	+		=	10
8	+		=	10
	+	1	=	10

Teaching notes

Counting

Counting on from the larger number in ones. By the end of the year encourage children to count in appropriate increments such as 2's. Use 100 squares and number lines to assist counting. Ensure children are counting once for each number 'jump'.

Adding

Children begin making groups of ten using base 10, cubes and ten frames. By the end of the year children are able to swap ten lots of ones for one lot of ten.

Children physically represent 2 digit numbers using base 10 and place value grids.

Number facts

Children can retrieve number bonds and facts from memory. Such as, $2+1=3$, $2+2=4$, $2+3=5$ etc.

By the end of the year children to have a good understanding of number bonds to 10 as well as related facts to 100. E.g. $2 + 8 = 10$ therefore $20 + 80 = 100$.

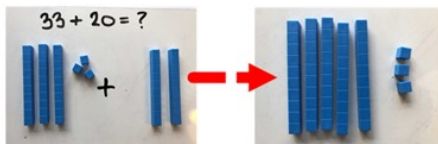
Addition



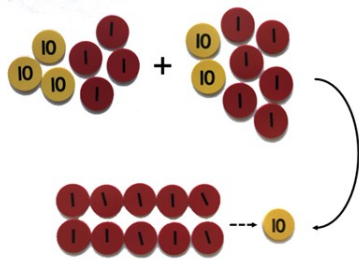
Year 2

Concrete

Use Base 10 to construct numbers using tens and ones..



Below shows the exchange of 10 ones to 1 lot of ten.



Counting on using ones before counting on tens.

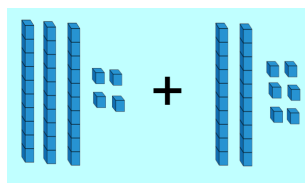
When using PV counters or Base 10 make sure to exchange 10 ones for one lot of ten before adding tens.

Pictorial

Children begin to partition numbers into tens and ones to support additions involving 2 digit numbers.

Tens	Ones

Combine the lots of ones, exchange if necessary, then combine the lots of tens.



34 + 26 you would first combine the 6 and 4 to make one lot of 10. Then combine the 3 lots of ten, 2 lots of ten and the exchanged 10.

Abstract

$$34 + 26 = ?$$

Begin with number sentences that don't require an exchange of ones. 35 + 2, 44 + 12, 63 + 24.

Progress onto additions that do require the exchange of ones. 35 + 5, 47 + 4, 56 + 17.

Using to expanded written method with column headings of Tens and Ones. Start with no exchange of ones.

Move onto questions involving some exchanges. Showcase the calculation at each stage by writing it next to the workings.

$$(4 + 6 = 10)$$
$$(30 + 20 = 50)$$

By the end of the year children should have been exposed to compact method but will continue to use expanded method

	T	O
	3	4
+	2	6
<hr/>		
	1	0
	5	0
<hr/>		
	6	0

	3	4
+	2	6
<hr/>		
	6	0

Teaching notes

Adding

Once place value understanding is secure children will begin to partition 2 digit numbers into tens and ones. Then using their counting strategies to find the total number of ones, exchange if required before find the total number of tens. By the end of year 2 children should be using number bonds and near number facts to short-cut simple calculations mentally. For example, 45 + 60. Children should know 4 + 6 = 10, therefore 40 + 60 = 100, then add the remaining 5. This should be encouraged over expanded or formal column methods when the child is displaying good understanding of the concept.

Number facts

By the end of year 2 children should be able to recite from memory number bonds to 20 and associated number bonds to 100. They should know most number facts and near number facts to 20 including doubles and near doubles.

Addition



Year 3 & 4

Concrete

Use place value counters to make 2, 3 and 4 digit numbers to support understanding of abstract calculations.

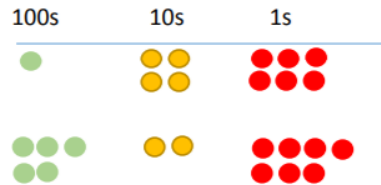


After creating the two numbers using Base 10, combining the ones, then the tens and so on, in that order, will find the total. This is used only to model process and shouldn't be relied upon.

Hundreds	Tens	Ones

Pictorial

Children can draw images to represent place value of a number to support addition of ones, then tens then hundreds etc.



This will progress to writing the digits rather than drawing images.

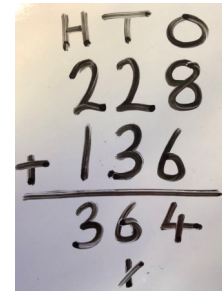
H	T	O
1	4	6
5	2	7

Children will begin to use bar models more extensively to show the 'parts' they know in order to support problem solving.

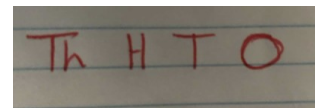


Abstract

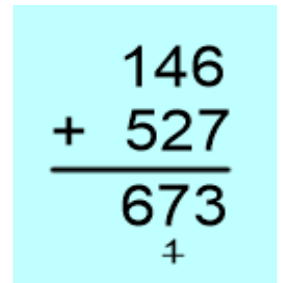
Numbers involved in calculation should initially only cross tens in a 3 digit addition. Moving on to crossing tens and hundreds.



Children can still use Th, H, T, O to label their columns.



Formal written method: use one digit per square calculate from ones first.

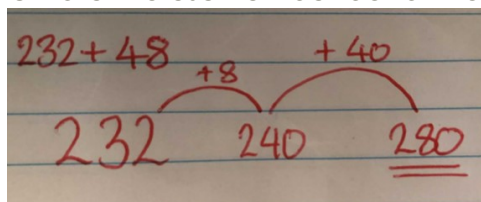


In year 4 children begin to use numbers up to 10,000.

Teaching notes

Adding

Children use strategies and understanding of place value, previously taught, to support them adding 3 digit numbers and ones, tens and hundreds. Children can use hand-drawn number lines to assist counting on. Encourage children to use number bond knowledge to draw more efficient 'jumps' along the line. For example,



Estimating

Children begin to use estimation to check answers as well as the inverse operation to ensure accuracy.

Addition



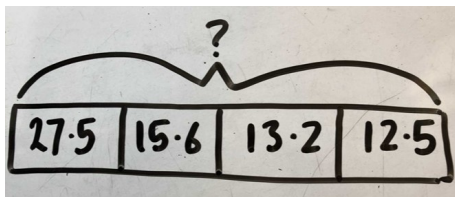
Year 5 & 6

Pictorial.

Base 10 can still be used to showcase examples of addition but should not be relied upon to calculate.

Thousands	Hundreds	Tens	Ones

Children should be encouraged to use visual structures, such as bar models, to layout known information. This is especially useful when dealing with complex word problems that don't have a clear number sentence.



Abstract

Building on the year 4 strategies now working with numbers up to 1 million. Ensure appropriate numbers are used when dealing with large numbers for the first time. Note, no crossing ten or carrying over is required.

Once conceptual understanding and confidence in the method is established you can begin to include carrying starting with ones, tens, hundreds etc.

Progressing to additions involving two decimal places. Money questions can place this into a real-life context.

	0	.	5	5	7	
	1	.	2	1	1	
+	0	.	2	0	2	
<hr/>						
	1	.	9	7	0	
						x

In year 6 explore additions to within ten million. Include questions to three decimal places.

Include problems that involve the addition of three numbers.

	TTh	Th	H	T	O
	4	3	2	0	1
	2	2	1	2	4
+	3	1	3	2	1
<hr/>					
	9	6	6	4	6

$$£132.52 + £213.83$$

	H	T	O	.	^{1/10}	^{1/100}
	1	3	2	.	5	2
+	2	1	3	.	8	3
<hr/>						
	3	4	6	.	3	5
<hr/>						
	1					

Teaching notes

Adding

Continue to use strategies laid out in year 3 & 4 including written formal method.

Model use of commas in larger numbers when writing answers. For example, 456,422.

When calculating using numbers with a decimal point ensure the decimal point is placed within the answer row before calculating.

Estimating

Children should be confidently estimating mentally to attempt to spot mistakes due to simple errors.

Rounding to the nearest 10, 100, 1000 etc. can be used to gain greater accuracy when estimating answers.

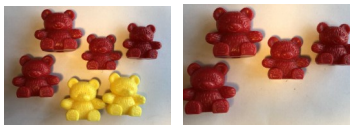
Subtraction



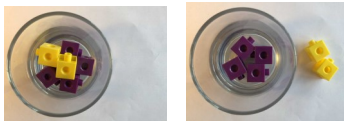
EYFS

Concrete

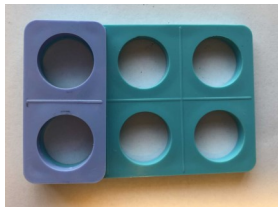
Use physical objects, bears, cubes, counters etc. to show how objects can be taken away.



There are 6 bears altogether, 2 bears are taken away, there are 4 bears left.



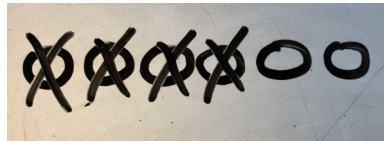
Containers can be used to demonstrate how many objects we 'have'. How many did we take out? How many are left?



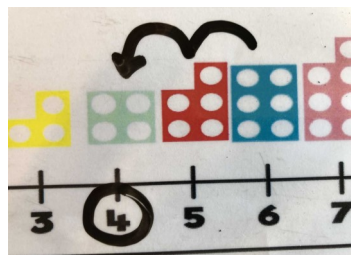
Placing the *subtrahend*¹ over the *minuend*² allows the child to see how many of the original amount are remaining when using Numicon.

Pictorial

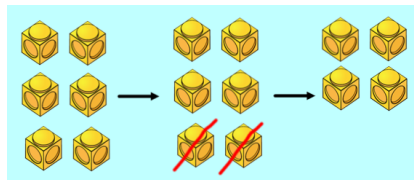
Cross out drawn objects to show what has been taken away.



Count back in ones from the whole to find how many are left.

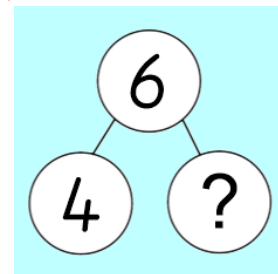


Below shows the process of taking away from a group. There are 6 cubes to begin with, 2 are taken away, there are 4 cubes left.



Abstract

Simple subtractions can be shown in a part, whole model to demonstrate that we know the whole and one of the parts and we are looking to find out the remaining part.



The number sentence would look like this..

$$6 - 2 = ?$$

Begin working with small numbers to 10. By the end of the year children should have strategies to solve problems to 20.

Using concrete resources and physically removing the part from the whole.

Counting back on a number line.

Drawing images and crossing out the amount indicated in the number sentence.

Using Numicon to show how many are left after a part has been taken from a whole.

Teaching notes

Counting

Using number lines to assist them children should be able to count back from 20 to 0. When counting out the 'part' from the whole ensure the child is using a 1:1 correspondence to maintain accuracy.

Subtracting

Children can use fingers to help them subtract. For example, $6 - 2 = ?$ They would show you 6 fingers, as they say the number sentence aloud 'six take away, one, two', physically folding one then a second finger down as they say 'one, two'.

¹ Subtrahend: A number to be subtracted from another, eg. $7 - 2 = 5$.

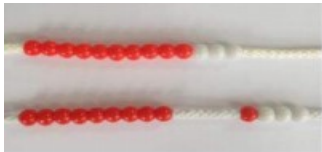
² Minuend: A number from which another number is to be subtracted from, eg. $7 - 2 = 5$.

Subtraction

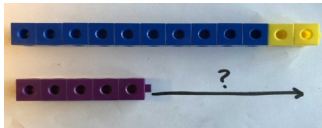
Year 1

Concrete

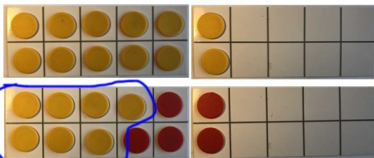
Making the whole number using a bead string. Move the beads back as you count backwards in ones to find the remaining part.



'Finding the difference' introduced. Counting up from the smaller number is advised. Use number bonds to assist counting.



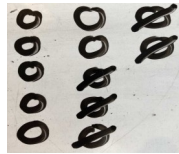
When subtracting numbers that cross 10, (initially using teen numbers only) use 10 frames to show concept physically.



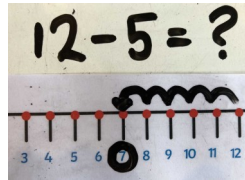
7 are left

Pictorial

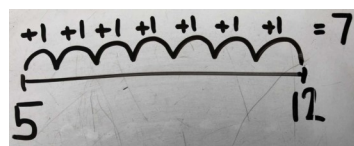
Children still physically cross off images to ensure conceptual understanding of 'taking away'.



Begin to use numbers that cross over ten. Children can still use number lines to help.

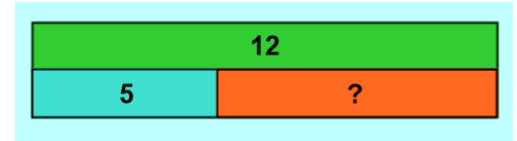


Children can begin to draw their own number lines, initially counting on ones to find the difference.

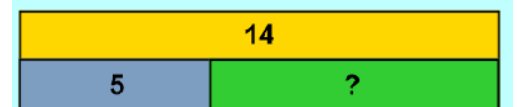


Abstract

Counting on to find the difference. Model the information from the number sentence $12 - 5 = ?$ using a bar. This supports concept understanding as it shows a clear size difference between the 5 and 12, objects can be used to support this or a grid.



Using number bonds to help mental calculation. Eg. $14 - 5 = ?$ First take away 4 to make 10, then take away the remaining 1 to make 9.



$$14 - 5 = ?$$
$$(-4, -1)$$

Teaching notes

Counting

When counting the remaining amount begin by counting in ones but by the end of the year encourage children to count more efficiently using 2's, 5's or 10's.

Subtracting

Pupils begin to explore missing number problems such as $12 - ? = 7$. Model drawing the information from the number sentence in a bar model or a part - whole model to see what we know and what we need to find out. This will show that we can use the number sentence $12 - 7 = ?$ to find the answer to the previous problem.

Checking

Check answers using the inverse operation. Eg. $12 - 5 = 7$ can be checked using $5 + 7 = 12$.

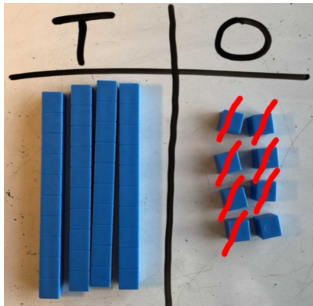


Subtraction

Year 2

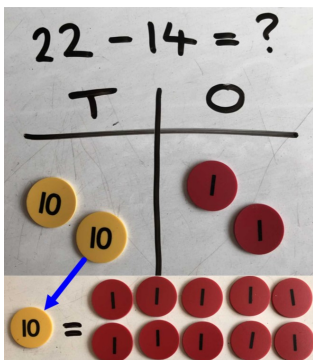
Concrete

Use Base 10 to physically build 2 digit numbers. Then remove or 'take away' the required amount.



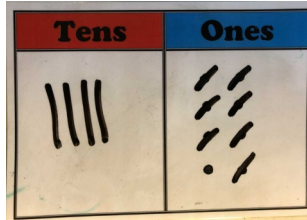
Begin by using calculations that don't require crossing tens.

Once children are confident with the concept, introduce calculations requiring the exchange of one ten to ten ones to make the subtraction possible.

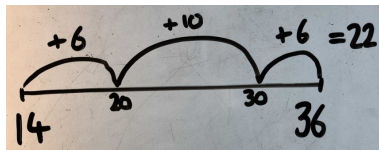


Pictorial

Children can draw 2 digit numbers in a similar way to using Base 10. Crossing off the number taken away to leave the remaining amount as the answer.



Encourage children to 'jump' to the next ten when using a number line to find the difference.

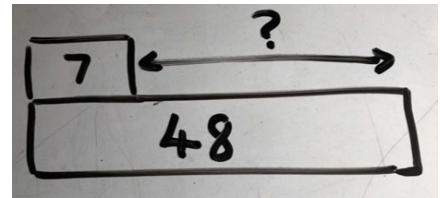


Use place value charts to support early column subtraction. Ensure the calculations chosen do not cross ten until concept is very secure.

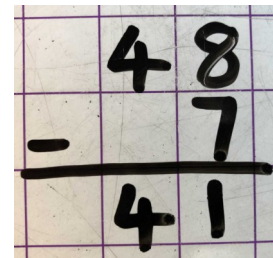
Tens	Ones
3	6
- 1	4
2	2

Abstract

Use bar models to visually show finding the 'difference'. Build on the year 1 method of using cube towers. The ? we want to find is 7 less than 48.

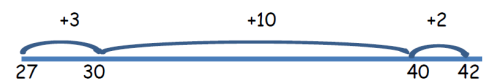


Use of column subtraction when no exchanges are required.



When faced with calculations such as $42 - 27 = ?$ children should use counting on method on a number line.

27 to 30 is 3, 30 to 40 is 10 and 40 to 42 is 2.



+3 and +10 and +2 is **15**.

Teaching notes

Counting

Counting back on a 100 square can be used when taking away a single digit number from a 2 digit number. Children should be encouraged to

Subtracting

Develop strategies from year 1, using a number line to take away and finding the difference by counting on from the smaller number.

Number bonds

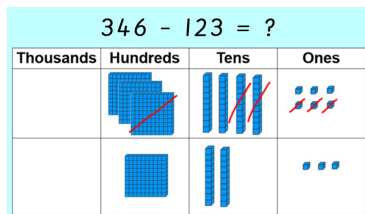
Knowledge of number bonds is critical to improve fluency when subtracting two 2 digit numbers. For example, spotting the difference or 'jump' of 3 from 27 to 30 is the first step to solving the example above.

Subtraction

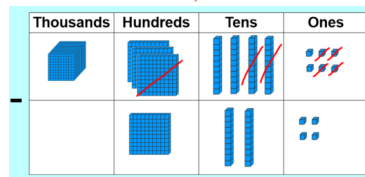
Year 3 & 4

Concrete

Use Base 10 to build larger numbers to support conceptual understanding. After building the whole, systematically 'take away' the part to leave the remaining part, showing the answer.



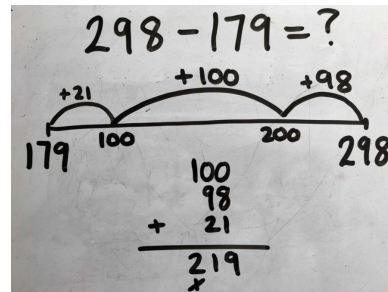
In year 3 calculations should involve numbers up to 1000.



Model laying out the subtraction practically to support children to visualise what they are doing when calculating.

Pictorial

Counting on from the larger number.



In year 4 children progress onto calculations involving numbers with two decimal places in context, for example:

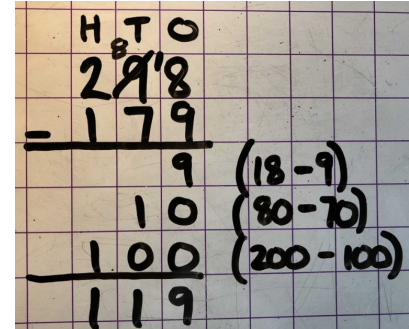
$$£213.83 - £183.51$$

Use formal column method to layout and solve.

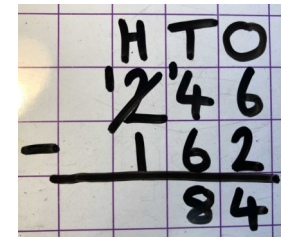
	H	T	O	•	t	h
	2	1	3	•	8	3
-	1	8	3	•	5	1
	0	3	0	•	3	2

Abstract

In year 3 children learn how to exchange in formal written method. Note the H T O place value headers to help layout of calculation. Begin by writing each step below until understanding is secure.



By the end of year 3 children should be using compact method to subtract, including exchanges.



Teaching notes

Subtracting

Model appropriate use of calculation choice. For example: 256 - 4 or 256 - 25 should be approached using mental strategies to encourage fluency. However, more complex subtractions should be tackled practically, initially, before moving to formal column methods, 256 - 127 for example.

In year 4 children should be confident in their choice of approach and understand why it is the most effective way for them to solve a problem. Ensure discrete teaching of mental strategies alongside written methods and promote the use of scrap paper notes to support informal written methods such as number lines.

When using formal column method ensure subtracting begins from the least significant figure, the ones. Use appropriate numbers in calculations where exchanging tens isn't required to ensure conceptual understanding. Once the child has a strong understanding of the principals involved more complicated subtractions involving exchanging should be attempted.

Inverse and estimating

Model using inverse operation to check answers. Check accuracy by rounding to the nearest 10 or 100 and estimating answer.

Subtraction

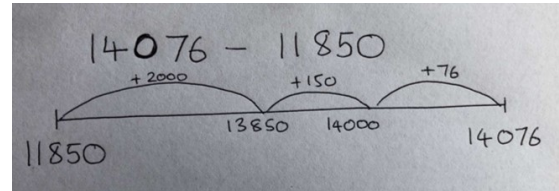
Year 5 & 6

Pictorial

When modelling and teaching mental strategies, refer to picturing a number line and either counting on or back.

$$\begin{aligned} ? &= 12462 - 2300 \\ &- 300 = 12162 \\ &- 2000 = \underline{\underline{10162}} \end{aligned}$$

For example: $14076 - 11850 = ?$



Abstract

Discrete teaching of more than one exchange. Model below how an exchange is needed and placed alongside a prior exchange in the case of the 'Tens' column.

Th	H	T	O
7	9	0	6
-	2	5	9
<hr/>			
5	3	0	8

Estimate

$$\begin{aligned} &7900 \\ - &2600 \\ \hline &5300 \end{aligned}$$

Estimation after rounding to the nearest, most appropriate number, 10, 100, 1000 etc. This too should be modelled and taught discretely encouraging children to be mindful of what they choose to round to for the sake of efficiency.

In year 6 model the use of brackets in multi-step problems identifying brackets as the initial step needed combining with additional written strategy.

$$633,465 + (745,676 - 325,534) = ?$$

Encourage children to choose appropriate calculation method. For example:

$$12 - 2.736$$

This can be tackled by counting on mentally or with notes on scrap paper referring to knowledge of number lines.

However, for the example below formal column method would be more efficient.

$$35.712 - 8.653$$

$$\begin{array}{r} 35.712 \\ - 8.653 \\ \hline 27.059 \end{array}$$

Teaching notes

Subtracting

In year 5 build on strategies from year 4 and start involving numbers of up to 100,000 before progressing to 1,000,000.

Formal written methods to now include 2 or more exchanges can occur.

Inverse and estimating

Modelling checking against estimates as a key part of the calculation process to ensure understanding. Encourage use of notepads or scrap paper to support mental methods. Check accuracy by rounding to the nearest 10 or 100 and estimating answer.

Multiplication



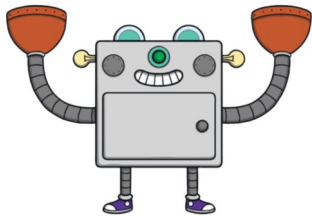
EYFS

Concrete

Use manipulatives to show how two lots of the same amount is called doubling.



They can use doubling machines both physically and digitally. Putting in two identical numbers and generating a 'doubled' total.

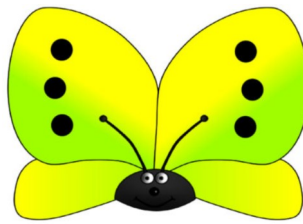


A similar learning opportunity can occur when using dice. Children should be encouraged to subitise and spot the parts 3 and 3. Later they may be able to subitise the whole without calculating.

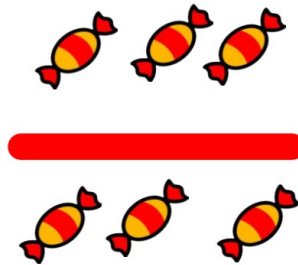


Pictorial

Children explore making doubles using symmetry, ladybirds and butterflies with spots on.



The key element here is understanding that doubling is adding the same number again.



Children should be move onto making another group of the same amount. E.g. I have three sweets, I double them to make three more, now I have six in total.

Abstract

Card matching games where children find identical 'doubles' before giving the 'doubled' total. Example: 5 and 5 make 10.



Children given questions like the one below verbally and towards the end of the year visually.

double 5 is ?

Children to leave year 1 with a clear understanding that doubling is adding the same number again.

Verbally introduce the idea that doubling the same number twice.

Teaching notes

Doubling

Children use a variety of manipulatives to generate doubles. They should have an idea that a double is the same number added again. For example: 3 add 3 equals 6, $2 + 2 = 4$.

Grouping

Expose children to the vocabulary of grouping and 'lots of'. For example: when using part-whole models. Children can spot if two groups are equal, having the exact same number of objects in each.

Multiplication

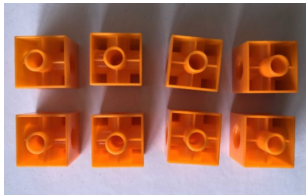


Year 1

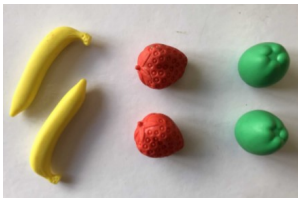
Concrete

Children build on their understanding of doubling from EYFS. Use manipulatives such as cubes to showcase doubles of an amount.

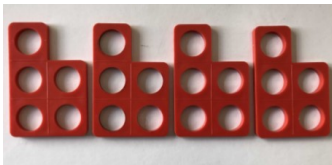
E.g. we have 4 cubes and we double it to have two groups of 4 cubes making 8 cubes in total.



Children continue learning about the idea of 'lots of' an amount. For example: I have three 'lots of' two, altogether I have six.



Children are introduced to the idea of repeated addition using equal groups. They learn to spot and then make equal groups and continue to use the term 'lots of' to describe.

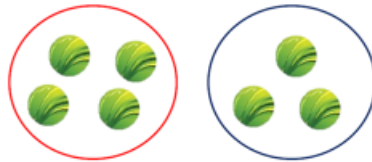


I have 4 lots of 5, altogether I have 20.

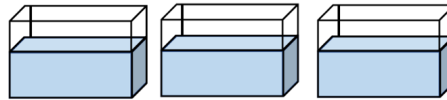
Pictorial

Children will be able to spot equal groups from images.

Are the groups equal?



Before moving onto making/drawing their own equal groups.



There are 3 fish in each tank, how many are there altogether?

4 There are 5 pots of pencils, each pot has 2 pencils inside.



problem
solving

How many pencils are there in total?

$$\square + \square + \square + \square + \square = \square$$

At this stage the children are still using repeated addition but being introduced to the language of multiplication including 'lots of' and times.

Abstract

How many lots of 2 make 8?



4 lots of 2 is ?

Children write repeated addition sentences until concrete in understanding is secure.

Introduced to the x (times symbol)

Linked to repeated addition 'x' number of times.

Teaching notes

Doubling



Children are introduced to times tables, 2's, 5's and 10's. Showcase these on a 100 square, counting stick as well as by using the manipulatives available to the children.

Show children arrays using contextual links such as: an egg tray or a box of chocolates.

We have 4 rows of 3 chocolates, 4 lots of '3 chocolates'. Link back to repeated addition.

Multiplication



Year 2

Concrete

Build on previous knowledge of arrays. Create and use arrays to practically explore commutative law.



Children can make arrays such as the one above showing 3 rows of 4 or 3 'lots of' 4.

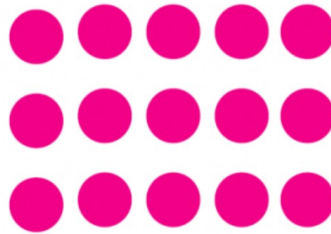


Using double sided counters is a good way to highlight that 3 'lots of' 4 is the same as 4 'lots of 3'.

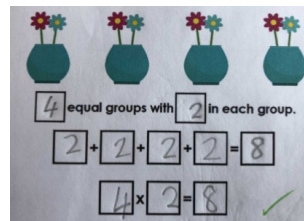


Pictorial

Children can begin to write multiplication number sentences to match arrays that are given or that they have made.



Use number lines to demonstrate how repeatedly adding the same amount links to multiplication.



Children can solve problems using repeated addition and linking that to simple 1 digit multiplication.

Abstract

Children write number sentences to show both repeated addition and multiplication.

$$5 + 5 + 5 + 5 = 20$$

$$5 \times 4 = 20$$

They solve problems that highlight the relationship between the two.

Teaching notes

Commutative law

The key learning point for year 2 is understanding how to read and make arrays. This highlights the law of commutativity. This states that regardless of the order of addition or multiplication the product is always the same. For example:

$$3 \times 4 = 4 \times 3$$

Recall and use multiplication facts for:

2, 3, 5, 10

Multiplication



Year 3 & 4

Concrete

Use base 10 to build each 'lot' before adding them together.

X	Tens	Ones

Model and discuss the efficiency of each approach referring to known skills such as doubling and doubling again etc.

$$215 \times 3 = ?$$

When faced with longer multiplications use place value grids to ensure clarity.

H	T	O

Children can then begin to layout calculations using formal method alongside the use of resources.

H	T	O
2	1	5
x		3
<hr/>		
6	4	5

Ensure the values are considered when setting questions.

Teaching notes

Grid method

Grid method is a short stepping stone towards written formal method. Using place value counters and building arrays leads into this stage.

Formal written method

Model formal written method in year 3. Encourage use only once understanding is secure using expanded method.

Pictorial

X	10	2
3		

Begin to display in an calculations in the form of grid method.

Abstract

X	3
10	30
2	6
<hr/>	
	36

Children use grid method to solve calculations involving a 2 digit and a 1 digit number.

For example: 12×3 or 34×6

H	T	O
1	4	3
x		6
<hr/>		
	1	8
2	4	0
<hr/>		
8	5	8

As children move into year 4 they begin to use multiples of ten based on previously known times table facts. For example: $30x$ or $40x$. Calculations include 3 digit by 1 digit multiplications.

Initially modelling and using an expanded formal written method.

H	T	O
1	4	3
x		6
<hr/>		
	8	5
8	5	8

Quickly moving onto compact formal written method.

Recall and use multiplication facts for:
all multiplication and division facts
up to 12×12

Multiplication



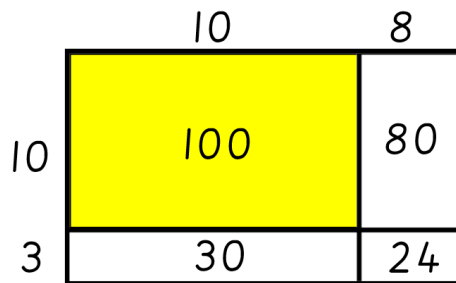
Year 5 & 6

Concrete & pictorial

In year 5 introduce multiplication where both factors can have 2 digits with one factor up to 4 digits. For example:

$$13 \times 18 = ?$$

Referring back to year 5 strategies to support transition from 1 digit factors to 2 digit. Ensure appropriate choice of numbers to highlight concept. Use of visual support showing the different rows within the grid method.



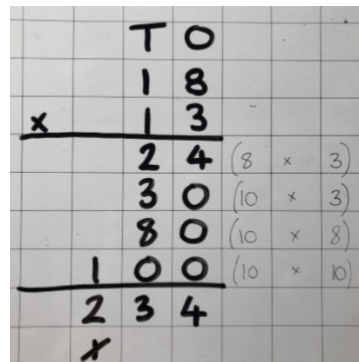
The use of manipulatives can be employed to support children and introduce a factor with an increased number of digits.



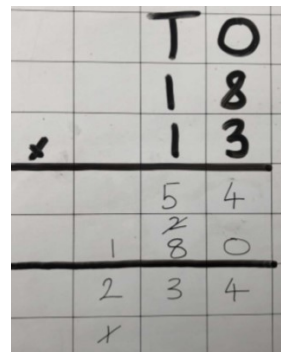
It should be noted that this is a stepping stone to ensure understanding and not to be regularly relied upon.

Abstract

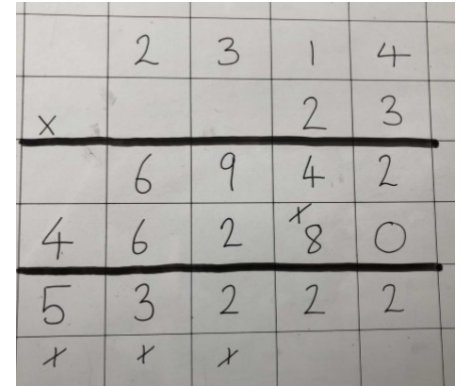
Once they have explored using the visual grids move onto expanded written method, modelling of noting steps to help with self checking. Monitor for solid knowledge of place value.



As soon as children can demonstrate competency using expanded written method they should be actively encouraged to move to formal written method.

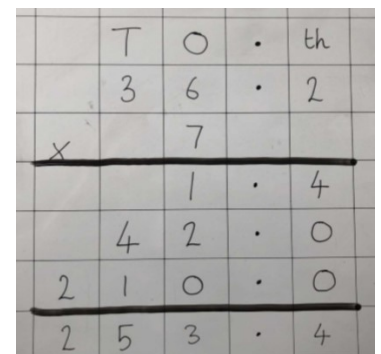


By year 6 most pupils should be using this method to set out calculations.



Pupils progress towards multiplying 4 digit numbers by 2 and 3 digit numbers using formal written method.

In year 6 the children will also multiply 4 digit numbers including one decimal point by 2 digit numbers. For example:

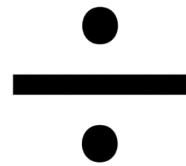


Teaching notes

Efficiency

Children should be taught both discreetly and explicitly to spot more efficient ways of calculating. Being aware of shortcuts such as: doubling and using $\times 10$, $\times 100$ etc knowledge. For example 2341×20 should not be done using written method as the numbers are all < 5 . Using pre-existing knowledge of place value and $\times 10$ this calculation can be done mentally or with simple jotting notes.

Division



EYFS

Concrete

The ELG states children should be able to solve problems including halving and sharing.



Start with the most basic idea of a half. Two parts that are the same.

Is it fair? Is it the same on both sides?

Develop this idea to include small amounts of objects such as shells.

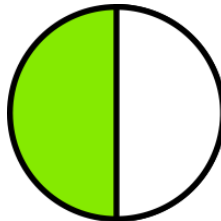


Children share the total or whole between two thus creating two equal parts.

By the end of EYFS children should be able to share an even numbered group of objects between two people.

Pictorial

As their understanding of sharing and halving increases expose them to visual representations of what a half looks like.



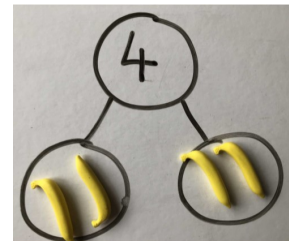
Some will be standard like the circle above but remember to include non standard images too. Especially for the higher ability.



By the Summer term children should be able to explore using manipulatives such as bar model kits, Numicon, cubes and bead strings to find a half of something.

Abstract

While not yet introducing the ÷ symbol children should have access to more formal representations.



Model the systematic approach to sharing equally.

Teaching notes

Halving

When teaching halving expose children to a wide variety of physical ways of making a half. For example, cutting string, folding paper, cutting cupcakes, tearing cotton wool, snapping spaghetti.

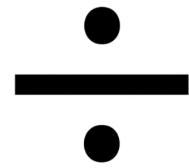
The concept can be further explored using balances (must be equal on both sides to create the balance) or through the use of water jugs.

It is important that children have opportunity to explore finding a half of a quantity of objects through sharing fairly.

The key elements to remember are both parts are the same and therefore equal.

Note the relationship between halving and doubling. Halving gets smaller, doubling gets bigger.

Division

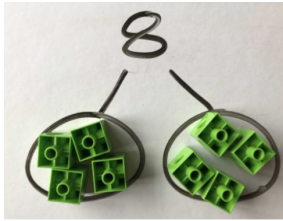


Year 1

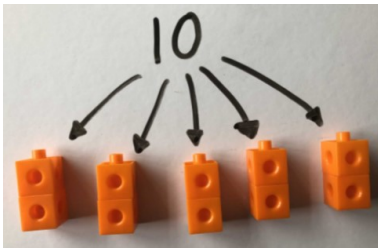
Concrete

Reinforce understanding of sharing and halving from EYFS.

Recognising and making halves followed by recognising and making equal groups.



Share equally and systematically initially using manipulatives before moving onto visual supports.

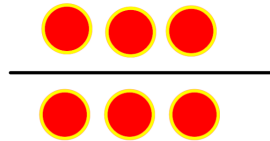


Model sharing of chunks or groups to show how many 'lots of' something make up a whole. Link to the inverse being multiplication.

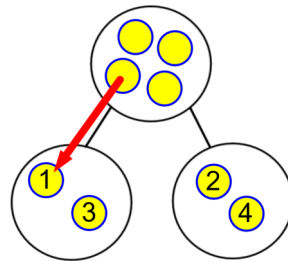
E.g. 5 'lots of' 2 makes 10 alongside 10 shared equally between 5 groups gives 2 in each group.

Pictorial

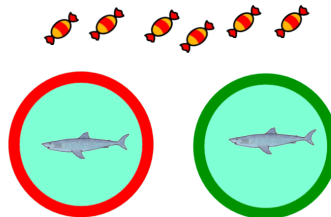
Children need to spot equal groups and understand, for now, that division only occurs when all the groups are equal.



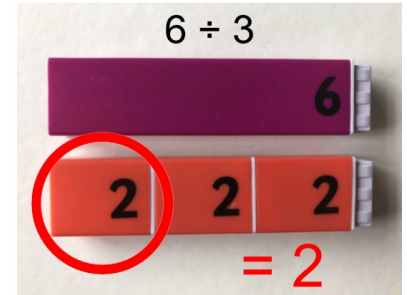
Use part-whole models to showcase sharing systematically.



Develop understanding using sharing or 'sugar' sharks. The idea being each shark must be fed equally or else they get grumpy! The idea of a 'remainder' can be introduced here and taught alongside odd and even.



Abstract



Model the use of manipulatives to support children in accessing \div .

The common misconception is children getting confused when writing the answer. The important language is the idea of finding the value of ONE PART (whilst ensuring all the parts are equal).

$$6 \div 2 = \square$$

$$6 \div 3 = \square$$

$$6 \div \square = 2$$

Model sharing using a part-whole model using manipulatives to help.

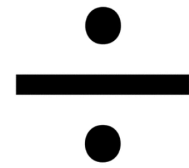
Teaching notes

Sharing systematically

Using 'sugar sharks' to give a 'real' reason as to why things must be equal and showcase the importance of being systematic. Use the sentence structure below to support children in articulating their sharing or dividing.

We had sweets, we shared them equally between sharks, each shark got .

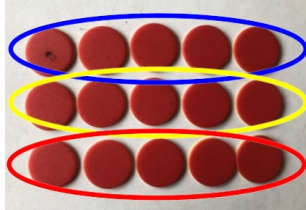
Division



Year 2

Concrete

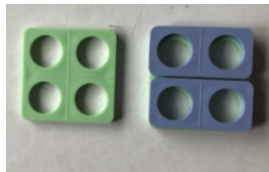
Children continue to use strategies taught in year 1 whilst beginning to use their knowledge of times tables to support problem solving, including short-cutting.



Use of arrays to highlight the link between division and multiplication. For example, 3 lots of 5 make 15 and 15 shared equally between 3 is 5.

$$3 \times 5 = 15$$

$$15 \div 3 = 5$$

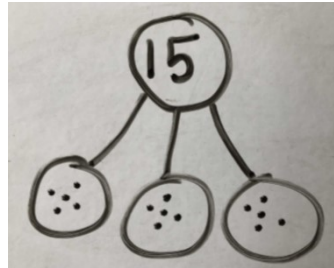


Use of Numicon to show how many lots of 2's 'fit' into a whole showcasing how the whole (4) is divided into 2 groups of 2.

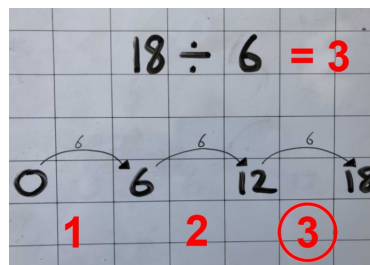
Pictorial

Children can now independently draw and use part-whole models to help visualise equal sharing. Alternatively they can use bar models.

The key to success here is being systematic as well as making clear legible marks.

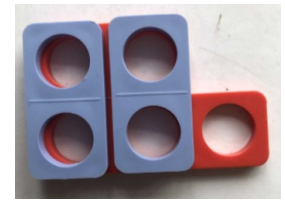


Model counting in parts up to the desired whole. For example...



The number of parts needed was 3. The answer is three because the whole can be shared into three equal parts.

Abstract



Children are introduced to the idea of a remainder using Numicon to visualise the fact that 5 cannot be shared equally into 2 parts.

There will be one left over.

Children should now be confident in identifying the number sentences that link division and multiplication.

Teaching notes

Inverse

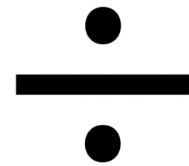
Children should be confident in using 2, 5 and 10 times table facts to support division calculations. They will develop their understanding of the inverse and the associated number facts.

Recall and use multiplication facts for:

2, 3, 5, 10

When appropriate children will begin to use the 3 times table as well as previously learned facts.

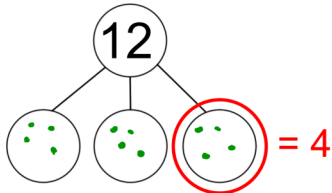
Division



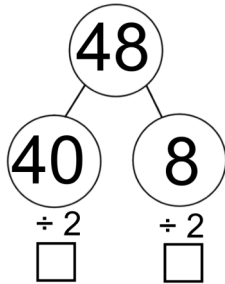
Year 3

Concrete & Pictorial

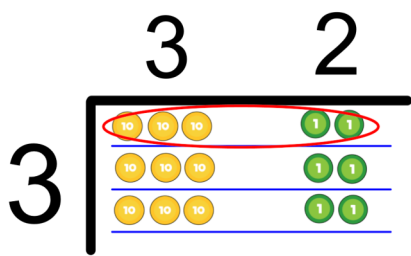
Initially using previously taught strategies to support conceptual understanding is fine.



Before quickly moving onto more complex problems. Breaking down 2 digit numbers into tens and ones to make calculating easier. This stage is a stepping stone and only works for multiples.



Moving forward children can begin to use informal written method. This can (and should) be done using place value counters to visually show what is happening in the calculation and why formal written methods work.



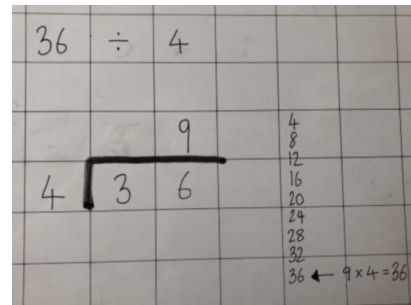
Teaching notes

'Bus stop method'

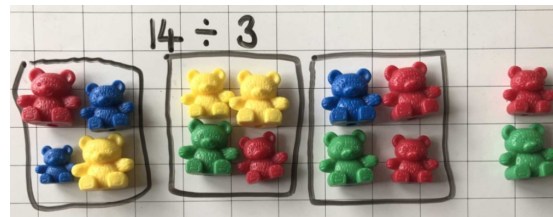
Very important that children have access to resources to support these initial stages of written formal method. Commonly children can access the simple divisions through replication but struggle later on when faced with long division often due to a lack of understanding from this stage.

Abstract

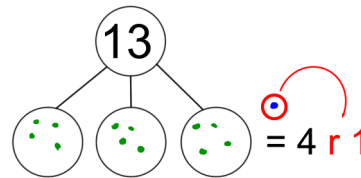
Children use the chunking method to divide 2 digit numbers by 1 digit numbers chosen from the list of known multiplication facts.



Layout of division including writing out of known multiplication table to support the 'chunking' phase.



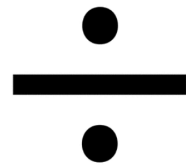
More exposure to calculations that include remainders. Build on previous knowledge of odd numbers and use manipulatives to highlight the concept.



Children can use previously taught strategies to support the initial understanding of remainders.

Recall and use multiplication facts for:
3, 4, 8

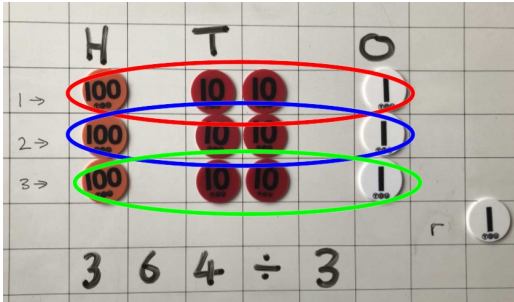
Division



Year 4

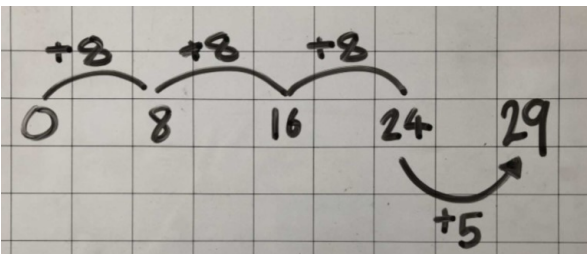
Concrete & Pictorial

Children continue to use strategies from year 3 as well as place value counters to support early calculations and conceptual understanding.



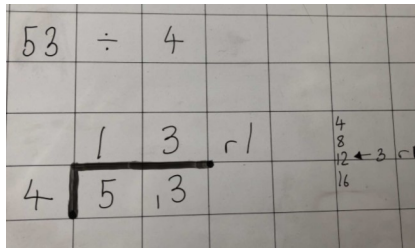
Children should move away from this stage only when they are secure using the written method and have a solid grasp of the concept.

Number lines can be used to count on in 'chunks' until the closest number to the whole is achieved before counting on to find the remainder.



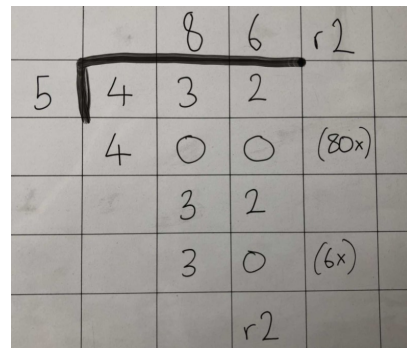
Abstract

Pupils continue to use short division strategy and knowledge of times tables and related division facts to solve simple fractions of amounts such as: find one eighth of twelve.

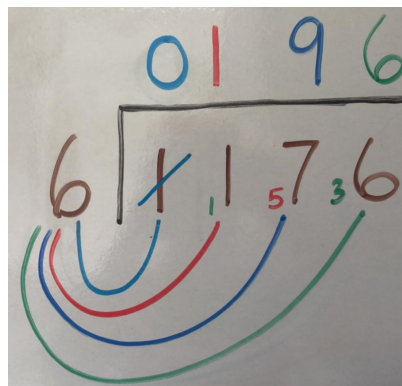


Use informal written method once concept is secure.

Build on year 3 method with 2 digit numbers and simple remainders.



Before moving to formal written method use expanded briefly when introducing 3 digit numbers with or without remainders.



3 digit number divided by a 1 digit without a remainder before progressing to 3 digit numbers with a single digit remainder.

Once children are confident in the process and have a strong conceptual understanding move onto formal written method as in the image shown to the left.

Using colours helps differentiate the different place value and whilst timely, does secure the understanding of the process.

Teaching notes

Place value

Important that the children have an idea of why long division works. Using place value counters to build the dividend¹. Then by sharing the hundreds, tens and ones between the number of the divisor² you can find the quotient³. When one of the values is not a multiple of the divisor you must exchange. For example exchanging the 'left over' ten for ten ones and carrying them over to the existing ones in order to begin the sharing process again.

Recall and use multiplication facts for:
all multiplication and division facts
up to 12 x 12

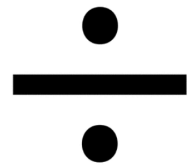
¹ Dividend: the number that is being divided.

² Divisor: the number that you are dividing by.

³ Quotient: the answer to the division problem. E.g. six divided by two gives a quotient of three.

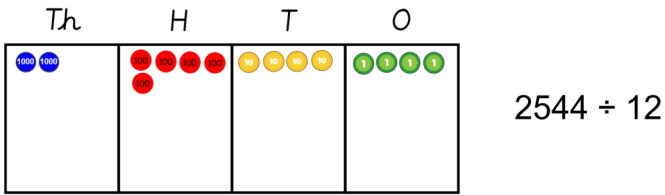


Division



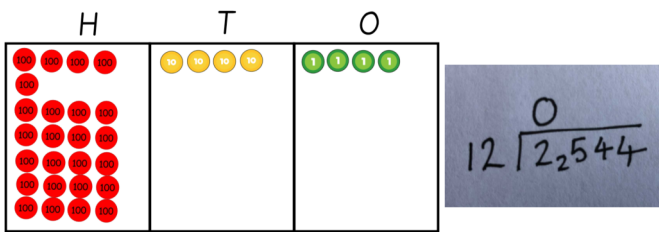
Year 5 & 6

Concrete & pictorial



How many groups of 12 thousand do we have?

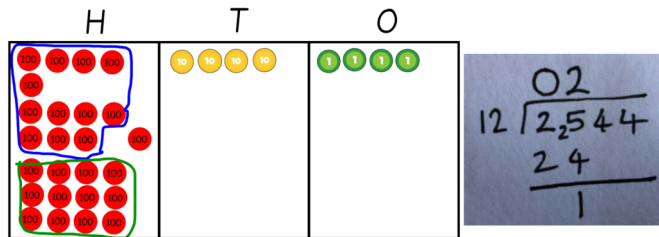
Exchange 2 thousand for 20 hundreds.



How many groups of 12 are there in 25 hundreds?

2 groups. Clearly draw around them.

We have grouped 24 hundreds leaving 1 hundred remaining.

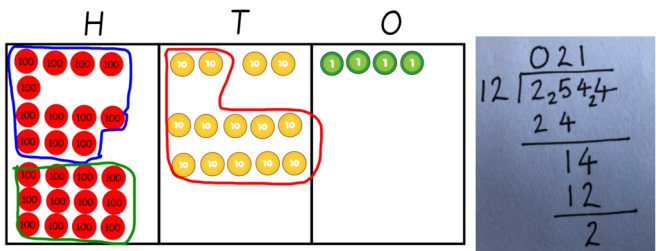


Exchange the one hundred for 10 tens so that we have 14 tens.

How many groups of 12 are in 14?

Clearly draw around the 12 showing 2 tens are remaining.

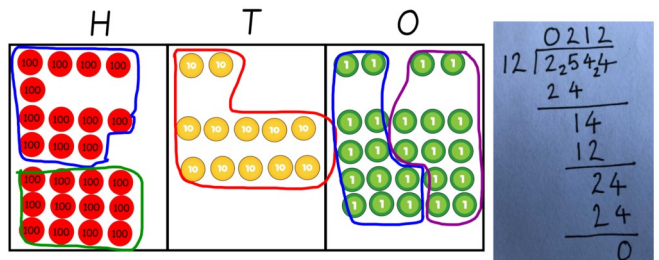
1 remainder 2.



Exchange the 2 tens for 20 ones giving 24 ones.

How many groups of 12 are in 24?

Clearly draw around each group of 12 giving the answer of 2.

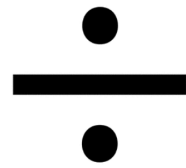


Teaching notes

Modelling

This is a very short step to ensure everyone has an understanding of what is really happening when dividing.

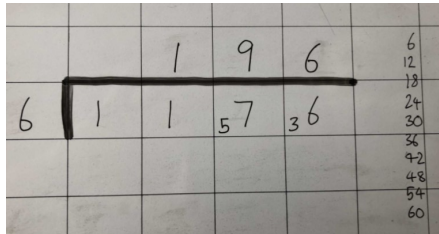
Division



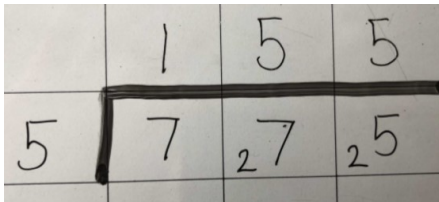
Year 5 & 6

Abstract

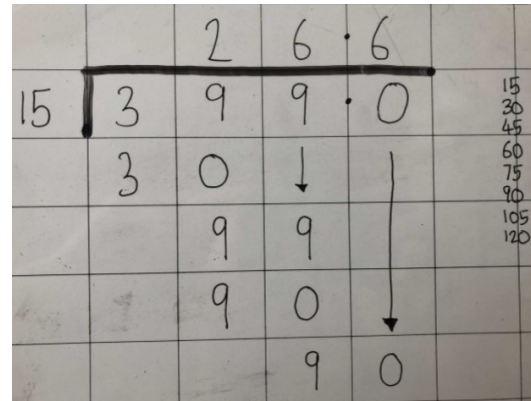
Children develop the use of short division method from year 4.



Continuing to write the times table alongside.

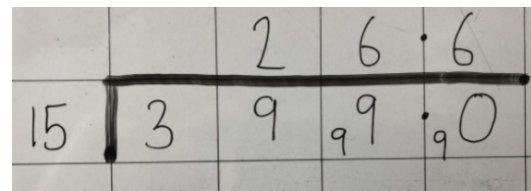


Note the choice of dividend and divisor when increasing the number of digits from 3 to 4.



Showcase long division built on concrete understanding (see previous page). Model layout briefly before moving onto shorter format.

Note the 15 times table written down the side of the calculation.



Note decimal place retains it's position.

Teaching notes

Remainders & tips

Essential that children are aware of shortcuts and have the confidence to use their knowledge of number to speed up calculation. For example: there is no point in using bus stop method to calculate $91 \div 7$. 10×7 is 70 leaving 21 which is divisible by 7 therefore giving an answer of 13.

They should be aware of any division by a power of 10 and avoid any formal written method in calculating these.

Children can continue to write down the times table alongside the calculation.

Ensure lots of discussion and support children in securing their understanding of place value.

Look for shortcuts such as dividing by a power of 10.

In year 5 children should begin to show remainders as fractions and then decimals. For example: $1176 \div 6$ will leave a remainder of 3. In reality this is $\frac{3}{6}$ or a half, this will ultimately be written as a decimal 0.5.

Rules of divisibility

You can determine if a number has a remainder by using the rules of divisibility. For example: a number is divisible by 2 if the last digit is even. A number is divisible by 3 if the sum of all it's digits is in the 3 times table. A number is divisible by 4 if the last TWO digits are in the 4 times table.



Helpful links

www.ncetm.org.uk

www.classroomsecrets.co.uk

www.tes.com

www.visualpatterns.org

www.mathshubs.org.uk

www.whiterosemaths.com



Useful terms

Aggregation: adding two or more quantities together, inverse of partitioning.

Augmentation: one quantity that is increased by an amount, inverse of reduction.

Subtraction: Partitioning– separate a number into two parts

Difference– between two values

Reduction– taking away of an amount

Associative law: it doesn't matter how we group numbers the total will remain the same.

Distributive law: multiplication can be done in two parts, for example: 6×7 can be done by $(5 \times 7) + (1 \times 7)$.

Commutative law: to swap the numbers order and still get the same answer (in addition and multiplication).

Addends: the parts of an addition number sentence.

Minuend: a number from which another is subtracted.

Subtrahend: a number to be subtracted from another.

Dividend: the number that is to be divided or shared.

Divisor: the number that you are dividing by.

Quotient: the answer to a division problem.

Subitise: to recognise an amount of objects simply by viewing them.